increases with decreasing rotation rate and there appears to be no extinction limit at low rotation rate. With radiative loss, flame temperature drops at low rotation rate as a result of amplification of the surface radiative loss parameter S and a radiative quench limit exists at sufficiently low rotation rate. The existence of the blowoff and quench limits with respect to rotation rate for this problem is similar to those found previously in forced-flow stagnation flames,[1] as shown in Fig. 1. However, the maximum flame temperature is higher for the rotational case (7.4 at $\Omega = 156$ rad/s) compared to 7.2 at $a_f = 15$ 1/s for the forced case. This has consequences for the low oxygen limit, which is discussed next.

Using oxygen mass fraction and rotation rate, flammability boundaries are shown in Fig. 2. For the rotational case, it consists of a blowoff branch at high rotation rates and a radiative quench branch at low rotation rates. The merging point (C) between the two branches defines the critical oxygen limit below which the solid disk will not be flammable at any rotational rate. The shape and the physical interpretation of this boundary is similar to the one derived in a previous study of forced-flow stagnation diffusion flame,[1] which is also plotted in Fig. 2 for comparison. Despite the similarity of shape between the two boundaries, the critical oxygen mass fraction for the rotational case (as indicated by point C) is lower than that for the forced flow case (as indicated by point $C\prime$). This is consistent with the finding in Fig. 1 that the maximum flame temperature for the rotational case is higher.

In Ref. 2, the flammability boundary for buoyancy-induced stagnation diffusion flame was compared to that for the forced-flow stagnation diffusion flame. The two boundaries can be collapsed into one if a properly defined buoyant stretch rate is adopted and the critical low oxygen limits are essentially the same for the two cases. Thus, the critical oxygen limit for the rotational solid is lower than that for the solid in buoyant flow as well.

An additional remark can be made on the influence of gas (flame) radiation. The recent addition of flame radiation to the model in Ref. 1 indicates a slight shift of the quenching boundary.[6] But for solid fuel with high surface temperature, the surface radiative loss dominates and this modification is very modest. We expect that the comparison between the rotational case and the forced and buoyant stagnation flame cases will remain true when flame radiation is included.

Concluding Remarks

Diffusion flame adjacent to a large, rotating solid fuel disk in the absence of buoyant force is modeled and numerically solved in this work. Similar to the conventional stagnation diffusion flame problems, the temperature and species distributions of this rotational flame are found to be one dimensional.[5] But unlike the stagnation-point problems, where the flow is two dimensional, driven either by inertia (forced convection) or by buoyancy, the flow in this problem is three dimensional, driven by a rotating solid through gas viscosity.

With the inclusion of a one-step finite-rate gas-phase reaction and a surface radiative loss term, the model predicts the existence of a blowoff extinction limit at high rotation rate because of insufficient gas residence time and a quenching limit at low rotation rate because of excessive radiative loss. The existence of these limits and the general flame behavior are similar to those in stagnation-point diffusion flames studied previously, with rotation rate playing the role of stretch rate. However, the different flowfield produces an important quantitative difference: the critical low oxygen percentage that can sustain a solid-fuel diffusion flame in the rotational case is lower than those in the stagnation flame cases. This suggests that a slow-rotating solid in a quiescent ambient can be more flammable than a stationary solid in a flowing stream, a result that should have implications for fire safety consideration in spacecraft. An ongoing research project may provide the needed experimental data for this prediction.

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Orthogonalization of Measured Modes—Revisited

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Introduction

THE orthogonalization of measured modes has been treated by Targoff,[1] Baruch and Bar-Itzhack,[2] and others. In Ref. 2 a closed-form solution has been found by minimization of the mass weighted distance between the orthogonalized and measured modes (see also Refs. 3 and 4). Recently, Zhang and Zerva[5] introduced a method in which they obtained the solution by minimization of the nonweighted distance between the orthogonalized and the measured modes. Here arises the interesting question: what is the right way to measure the distance between two matrices? The question of distance between two matrices is directly connected with the notion of length of a vector. The concept of length of a vector in static or dynamic surroundings is treated in Refs. 6 and 7.

Generalized Orthogonalization

Following Refs. 6 and 7 the generalized distance between two vectors $\boldsymbol{V}$ and $\boldsymbol{Z}$ can be defined as

$$\Delta\ell^2 = (\boldsymbol{V} - \boldsymbol{Z})^t W(\boldsymbol{V} - \boldsymbol{Z}) = \left\| W^{\frac{1}{2}}(\boldsymbol{V} - \boldsymbol{Z})\right\| \tag{1}$$

where $\|\cdot\|$ symbolizes the Euclidean norm and W is a positive definite matrix. Equation (1) can be generalized also for matrices.

Let $W_1(n \times n)$ and $W_2(n \times n)$ be two positive definite symmetric matrices. The generalized distance between the measured and the orthogonalized mode shapes will be in respect to W_1, and the mode shapes will be orthogonalized in respect to W_2 so that

$$X^t W_2 X = I \tag{2}$$

where $X(n \times m)$ are the orthogonalized mode shapes, n are the degrees of freedom of the structure, and m is the number of measured mode shapes.

Before the minimization process, every measured mode shape must be normalized:

$$T_i = \tilde{T}_i(\tilde{T}_i^t W_2 \tilde{T}_i)^{-\frac{1}{2}} \qquad i = 1 - m \tag{3}$$

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where $\tilde{T}_i$ is the measured mode shape. The function to be minimized, including the constraint given in Eq. (2), can be written as follows:

$$g = \left\| W_1^{\frac{1}{2}}(X - T)\right\| + \tfrac{1}{2}\left| \lambda(X^t W_2 X - I)\right| \tag{4}$$

where $|\,.\,|$ stays for the sum of every equation in Eq. (2) multiplied by an unknown coefficient (see Ref. 2). Because of the symmetric constraint, the Lagrange matrix λ is symmetric. The partial differentiation of Eq. (4) in respect to any element of X yields

$$\frac{\partial g}{\partial X} = W_1(X - T) + W_2 X\lambda = 0 \tag{5}$$

Equations (2) and (5) yield

$$\lambda = X^t W_1 X - X^t W_1 T \tag{6}$$

But λ is symmetric; hence,

$$X^t W_1 T = T^t W_1 X \tag{7}$$

Following Ref. 5, we will look for a solution in the form

$$X = T(T^t W_1 T)^{-1} S \tag{8}$$

where $S(m \times m)$ is a positive definite symmetric matrix. From Eqs. (2) and (8) one obtains S and finally,

$$X = T(T^t W_1 T)^{-1}\left[(T^t W_1 T)^{-1} T^t W_2 T (T^t W_1 T)^{-1}\right]^{-\frac{1}{2}} \tag{9}$$

Special Cases

Case 1. This is the common full dynamic case:

$$W_1 = W_2 = M \tag{10}$$

where $M(n \times n)$ is the known mass matrix. Substitution into Eq. (9) yields

$$X = T(T^t M T)^{-\frac{1}{2}} \tag{11}$$

Equation (11) was obtained previously in a direct way[2] and is quite elegant. Equation (11) justifies the assumption of Eq. (8).

Case 2. This is the static dynamic case:

$$W_1 = I \qquad W_2 = M \tag{12}$$

Substitution into Eq. (9) yields

$$X_{\mathrm{SD}} = T(T^t T)^{-1}\left[(T^t T)^{-1} T^t M T (T^t T)^{-1}\right]^{-\frac{1}{2}} \tag{13}$$

Equation (13) is equivalent to the result obtained in Ref. 5. In case 2 the generalized distance is geometric[7] (static), and the orthogonalization is dynamic.

Case 3. This is the full static case:

$$W_1 = W_2 = I \tag{14}$$

Substitution into Eq. (9) yields

$$X_{\mathrm{ST}} = T(T^t T)^{-\frac{1}{2}} \tag{15}$$

Equation (15) may be of some interest despite the fact that the notion of mode shapes does not exist in the pure static case. In the static case the Maxwell–Betti reciprocal theorem[8] must be employed.[9–11]

Discussion

The important difference between cases 1 and 2 is in the employed generalized distances. The relative dynamic and static distances will be defined as follows:

$$D_D = \left[\frac{\left\| M^{\frac{1}{2}}(X - T)\right\|}{\left\| M^{\frac{1}{2}} T\right\|}\right]^{\frac{1}{2}} \qquad D_S = \left[\frac{\|(X - T)\|}{\|T\|}\right]^{\frac{1}{2}} \tag{16}$$

Table 1 Relative distances

Case	D_D	D_S
1	0.1605	0.1488
2	0.1690	0.1392

Equations (11), (13), and (16) are now employed in a example taken from Ref. 2 (Table 1).

As expected, D_D for case 1 is smaller than for case 2 and D_S is smaller for case 2 than for case 1. Now, the question is which case must be preferred. It seems that, for a dynamic structure, case 1 is more natural. Hence, case 1 is preferable.

If the stiffness matrix of a dynamic structure is known (or identified by using static measurements[10]), one can use measured natural frequencies and mode shapes to identify the unknown mass matrix.[12] In this case the orthogonalization must be with respect to the known stiffness matrix. The normalization of the individual mode shape is as follows[9]:

$$T_i = \omega_i \tilde{T}_i(\tilde{T}_i^t K \tilde{T}_i)^{-\frac{1}{2}} \qquad i = 1 - m \tag{17}$$

where $K(n \times n)$ is the known stiffness matrix and ω_i is the measured natural frequency. The minimization process[9] yields

$$X_K = T\Omega(\Omega T^t K T \Omega)^{-\frac{1}{2}}\Omega \tag{18}$$

where $X_K(n \times m)$ are the orthogonalized mode shapes and $\Omega(m \times m)$ is the diagonal matrix of the measured natural frequencies. The elastic modes can be easily isolated from the rigid body modes[9] and, in contrast to the case in Ref. 5, one can include a credibility matrix in the closed-form equations (11) and (18).

Conclusions

The logical distance between two matrices in a dynamic structure is the distance weighted by the mass or the stiffness matrix. Hence, Eqs. (11) and (18) present the natural orthogonalization of measured modes.

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